Seat No. : _______________

N14-113

November-2014

B.Sc., Sem.-V (CBCS)

STA : 302 – Statistics

(Statistical Inference & Design of Experiments – I)

Time : 3 Hours] **[Max. Marks : 70**

Instructions : (1) **All** questions are of equal marks.

(2) Scientific calculator is permitted to use statistical table will be supplied on request.

1. (a) Define statistic and estimator. Exemplify the statement : "A statistic is not necessary to be an estimator for estimating the parameter θ of the distribution having pdf f(*x*, θ)". Also give an example of a statistic which is an estimator of parameter θ. Identify the statistics from the given below with reason :

(i) $T_1 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$ (ii) $T_2 = \sum_{i=1}^{n} (x_i - \bar{x})^2$

(iii) $T_3 = \bar{x}$ (iv) $T_4 = X_{(1)}$

OR

Discuss the general method of estimating an unknown parameter by interval estimation.

(b) Let $X_1, X_2, \ldots X_n$ be a random sample from normal distribution $N(\mu, \sigma^2)$, μ is known. Obtain (1 – α)100% confidence interval for σ^2. Find its expected length.

OR

Let $X_1, X_2, \ldots X_n$ be a random sample from the distribution having pdf $f(x, \theta) = \frac{1}{\theta} e^{-x/\theta}$, $x > 0$, $\theta > 0$. Obtain (1 – α) 100% confidence interval for θ. Find its expected length.

2. (a) State and prove Fisher-Neyman factorization theorem for discrete case to obtain a sufficient statistic.

OR

Stating clearly the regularity conditions establish Cramer-Rao inequality.

(b) Let X_1 and X_2 are two independent Bernoulli variates with parameter θ, 0 < θ < 1. Show that $T = X_1 + 2X_2$ is sufficient for θ. Also obtain Fisher information contained in Statistic T.

OR

Let $X_1, X_2, \ldots, X_n$ be a random sample from exponential distribution with mean θ, $\theta > 0$. Construct Rao-Blackwell statistic for θ and state its variance.

3. (a) What is maximum likelihood principle ? Discuss the method of scoring to obtain MLE with suitable example.

OR

Discuss the method of moments to obtain estimator of the parameter(s) of the distribution. Is this estimator of the parameter θ always sufficient ? Justify your answer.

(b) Let $X_1, X_2, \ldots X_n$ be a random sample from distribution having pdf

$$f(x; \mu, \sigma) = \frac{1}{\sigma} \exp\left(-\left(\frac{x-\mu}{\sigma}\right)\right); \; x \geq \mu, \sigma > 0.$$

(i) Obtain moment estimators for (μ, σ)

(ii) Obtain MLE for (μ, σ)

OR

Let $X_1, X_2, \ldots X_n$ be a random sample from exponential distribution with mean $1/\theta$, $\theta > 0$. Obtain MLE of θ. check whether it is (i) Consistent (ii) Unbiased for θ.

4. (a) Discuss fully one way analysis of variance technique.

OR

Discuss fully two way analysis of variance technique.

(b) Discuss how you use randomization, replication and local control in completely randomized design.

OR

Consider a completely randomized design for m treatments over n unit with the treatment replications $n_1, n_2, \ldots n_m$ times respectively. Discuss how do you test the hypothesis $H : \mu_1 = \mu_2 = \ldots = \mu_m$, where μ is the mean of i^{th} treatment.

5. Answer the following :

(i) State two unbiased estimators for the mean of the Poisson distribution.

(ii) Define consistent estimator.

(iii) Let $X_1, X_2, \ldots X_n$ be a random sample from Poisson distribution with mean $\lambda > 0$. $T = \left(1 - \frac{1}{n}\right)^{\sum_{i=1}^{n} Xi}$ is an unbiased estimator for $e^{-\lambda}$. What is the Cramer-Rao lower bound for the estimator T, if the Fisher information contained in the sample is n/λ ?

(iv) Give examples of : (1) biased MLE and (2) unbiased MLE.

(v) When completely randomized design is not advisable to use.

(vi) State the distributions for which method of moments is (i) applicable (ii) not applicable.